Microwave Measurement of Surface Impedance of High-Tc Superconductor

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ABSTRACT

Perturbation formulas for a TE_{011} mode dielectric resonator and for a TE_{011} mode circular cavity are derived to determine surface impedance Z_s ($=R_s+jX_s$) of superconductors from measured values of resonant frequency and unloaded Q. The Z_s values for a YBCO plate with a diameter of 30 mm were measured at 8.7, 10.4, and 23.7 GHz using these formulas. The measured results are presented as functions of temperature.

I. INTRODUCTION

In order to investigate the conduction mechanism and electronic application of high-T_c superconductors, it is important to measure the surface impedance $Z_s=R_s+jX_s$ in microwave and millimeter-wave range, where R_s is the surface resistance and X_s is the surface reactance. An conclusion derived theoretically from the two-fluid model is that R_s is proportional to ω^2 and X_s is proportional to ω [1]. For the frequency dependence of R_s, many experimental reports have been presented recently [2]-[5], and they have suggested the validity of the theory. On the other hand, experiments for X_s have been performed little so far.

In this paper, two measurement methods are used to determine the frequency and temperature dependences of X_s as well as R_s; one is a TE_{011} mode dielectric resonator method applicable in the frequency range of 5 to 20 GHz and the other is a TE_{011} mode cavity resonator method applicable in the range of 15 to 100 GHz when we use a plate sample of 30 mm in diameter. We first derive perturbation formulas for these resonators. Then we determine the R_s and X_s values of a plate of $YBa_2Cu_3O_{7-\delta}$, which is abbreviated as YBCO, from the measured values of the resonant frequency f_0 and the unloaded Q, Q_u for these resonators as a function of temperature.

II. SURFACE IMPEDANCE Z_s

Generally, the surface impedance Z_s is defined by the ratio of the electric field E_t to the magnetic field H_t tangential to a conductor surface;

$$Z_s = \frac{E_t}{H_t} = R_s + jX_s = \frac{j\omega\mu}{\gamma} \tag{1}$$

where γ is the propagation constant in the conductor and $\mu=\mu_0 = 4\pi\times10^{-7}$ H/m.

In a normal state for $T>T_c$, where T_c is the critical temperature of superconductor, γ is given by

$$\gamma = \sqrt{j\omega\mu\sigma} = \frac{1}{\delta} + j\frac{1}{\delta} \tag{2}$$

with

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad \sigma = \frac{\overline{\sigma}\sigma_0}{1+\alpha(T-293)} \tag{3}$$

where δ is the skin depth, σ is the conductivity, $\sigma_0=58\times10^6$ S/m for standard copper, $\overline{\sigma}$ is the relative conductivity, and α is the temperature coefficient of resistivity. Substitution of (2) into (1) gives

$$Z_s = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}} \tag{4}$$

In a superconducting state for $T<T_c$, on the other hand, γ is given by

$$\gamma = \sqrt{\frac{1}{\lambda^2} + j\frac{1}{\delta^2}} \tag{5}$$

according to two-fluid model [1], where λ is the penetration depth. In this case, substitution of (5) into (1) yields

$$R_s = \frac{\omega\mu\lambda}{\sqrt{2}}\sqrt{\frac{\sqrt{1+4(\lambda/\delta)^4}-1}{1+4(\lambda/\delta)^4}} \tag{6}$$

$$\approx \frac{\mu\sigma\lambda^3}{2}\omega^2 \qquad (\lambda \ll \delta) \tag{7}$$

$$X_s = \frac{\omega\mu\lambda}{\sqrt{2}}\sqrt{\frac{\sqrt{1+4(\lambda/\delta)^4}+1}{1+4(\lambda/\delta)^4}} \tag{8}$$

$$\approx \mu\lambda\omega \qquad (\lambda \ll \delta) \tag{9}$$

From (6) and (8), we also obtain

$$\delta = \frac{R_s^2 + X_s^2}{\omega\mu\sqrt{R_s X_s}} \qquad \lambda = \frac{R_s^2 + X_s^2}{\omega\mu\sqrt{X_s^2 - R_s^2}} \tag{10}$$

III. MEASUREMENT METHOD OF Z_s

Fig. 1(a) shows a TE_{011} mode dielectric rod resonator

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placed between a perfect-conductor plate with $Z_{sm}=R_{sm}+jX_{sm}=0$ and a conductor plate with a finite Z_s value, such as superconductor or metal. A dielectric rod having relative permittivity ε_r, diameter D, and length L is assumed to be lossless. Fig. 1(b) shows a TE_{011} mode circular cavity which contains a perfect-conductor cylinder and plate with $Z_{sm}=0$ and a conductor plate with a finite Z_s value. Fig. 1(c) shows an equivalent circuit for these resonators, from which the resonance condition is given by

$$Z_s + jZ_\beta \tan \beta L = 0 \tag{11}$$

where $Z_\beta=\omega\mu/\beta$ and β are the characteristic impedance and phase constant in the dielectric. Then, taking the first order approximation of (11) and using a perturbational quantity of complex frequency $\Delta\dot{\omega}/\omega$, we can derive a perturbation formula for Z_s; that is,

$$Z_s = 960\pi^2 \left(\frac{L}{\lambda_0}\right)^3 \frac{\varepsilon_r + W}{1 + W}\left(-j\frac{\Delta\dot{\omega}}{\omega}\right) \tag{12}$$

where $\lambda_0=c/f_0$, c is the light velocity in vacuum, W is the constant given by calculation [6], and $\Delta\dot{\omega}/\omega$ is given by

$$\frac{\Delta\dot{\omega}}{\omega} = \frac{\Delta f}{f} + j\frac{1}{2Q_c} \qquad \Delta f = f_0 - f \tag{13}$$

where f is the resonant frequency when $Z_s=0$, f_0 and Q_c are the resonant frequency and Q due to the conductor loss when $Z_s \neq 0$.

Similarly, we can derive the following perturbation formula for the cavity in Fig. 1(b):

$$Z_s = 960\pi^2 \left(\frac{L}{\lambda_0}\right)^3 \left(-j\frac{\Delta\dot{\omega}}{\omega}\right) \tag{14}$$

Since the relation $R_s=X_s$ is obtained from (4) in the normal state, particularly, the resonant frequency for the loss-free case is given by

$$f = \frac{2Q_c}{2Q_c - 1} f_0 \tag{15}$$

which is derived from either (12) and (13) or (13) and (14).

In an actual resonator, both loss-tangent of dielectric $\tan\delta$ and R_{sm} are not zero. In order to obtain Q_c, therefore, it is necessary to remove the effect of these losses from a measured Q_u value; that is,

$$\frac{1}{Q_c} = \frac{1}{Q_u} - \frac{R_{sm}}{480\pi^2}\left(\frac{\lambda_0}{L}\right)^3 \frac{1+W}{\varepsilon_r+W} - \frac{\tan\delta}{1+W/\varepsilon_r} \tag{16}$$

for the dielectric resonator and

$$\frac{1}{Q_c} = \frac{1}{Q_u} - \frac{R_{sm}}{480\pi^2}\left(\frac{\lambda_0}{L}\right)^3 - \frac{(3.8317)^2 R_{sm}}{60\pi^4}\left(\frac{\lambda_0}{D}\right)^3 \tag{17}$$

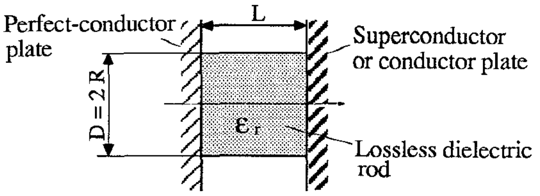
for the cavity.

The values of ε_r, $\tan\delta$, and R_{sm} required in (16) can be determined by measurement, following the procedure described in [6]. We can also determine the R_{sm} value required in (17) from a Q_u value measured for a TE_{011} mode copper cavity.

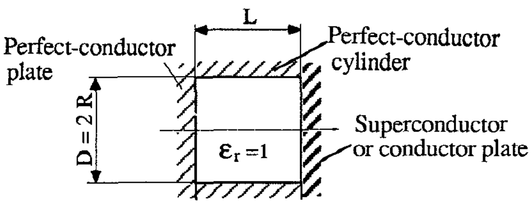
IV. MEASURED RESULTS

A. Dielectric resonator method

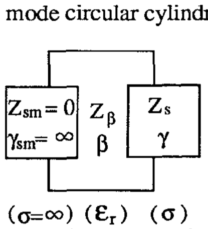
To determine the values of ε_r, $\tan\delta$, and R_{sm} in advance [6], we use a TE_{011} mode dielectric resonator of length L_1 and a TE_{014} mode dielectric resonator of length $L_4=4L_1$, cut from a $(Z_rS_n)T_iO_4$ ceramic rod of diameter D=7.3 mm (the coefficient of the thermal linear expansion τ_d=6.5 ppm/K, Murata Mfg. Co., Ltd.), as shown in Fig. 2. Each of these resonators is placed between two copper plates of diameter d_0=30 mm. Fig. 3 shows the experimental apparatus used actually, which is set in a



(a) TE_{011} mode dielectric rod resonator.



(b) TE_{011} mode circular cylindrical cavity.



(c) Equivalent circuit.

Fig. 1. Analytical model.

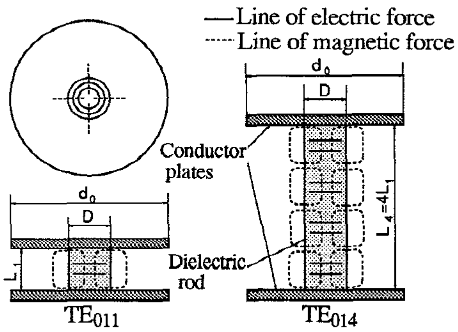


Fig. 2. TE_{011} and TE_{014} mode dielectric rod resonators placed between two parallel conductor plates.

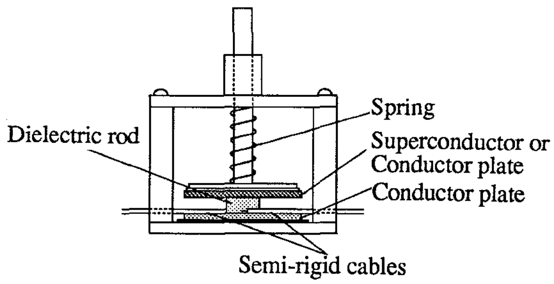


Fig. 3. Experimental apparatus.

He-gas closed loop refrigerator. The temperature dependence of ε_r, $\tan\delta$, and R_{sm} measured at 10.4 GHz is shown in Figs. 4 and 5. The solid curve in Fig. 5 indicates the R_{sm} value calculated from (3) and (4), using $\overline{\sigma}$=0.8 and α=0.00393 for copper. It is found from this result that the calculation is effective in the temperature range below 100 K.

Then we replace the upper copper plate by a YBCO plate of d_0=30 mm in the TE_{011} mode dielectric resonator, as shown in Fig. 3. Fig. 6 shows the measured result of Q_u in this case (YBCO+Cu), together with that for two copper plates (Cu+Cu). The Q_c values can be obtained from these Q_u values using (16). Similarly, the measured results of f_0 are shown in Fig. 7 by the closed and open squares, respectively. The solid curves A and B indicate ones estimated for the lossless conductors from (15) and (16) on condition that R_s=X_s, using the values of ε_r, $\tan\delta$, R_{sm}, Q_u, and f_0 given in Figs. 4 to 7. Since copper is always in the normal state with R_s=X_s, the curve A has not a discontinuity in all range of T. On the other hand, the curve B has a discontinuity at T=T_c=92 K, because the YBCO plate is in the superconducting state with $R_s \neq X_s$ at T<92 K. The difference between the curves A and B near room temperature can be explained to be due to the uncertain contacts which occur when the dielectric rod is set between two plates. This fact has been confirmed by the repeat test of setting the TE_{011} mode dielectric resonator near room temperature, as shown in Fig. 8. There is the amount of scatter below 100 MHz in the differences between the open and closed dots, which covers 9.3 MHz indicated in Fig. 7. Therefore, if the same condition of contact is realized, we can shift the solid curve A into the dot curve A' by 9.3 MHz. Thus the value

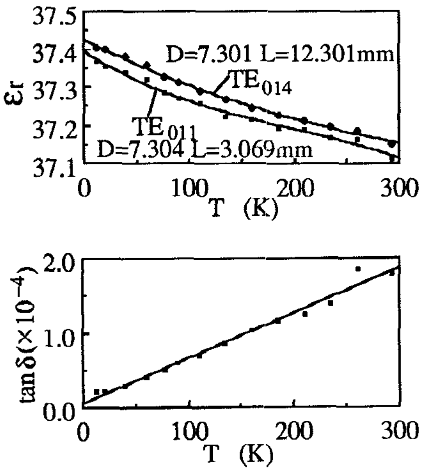


Fig. 4. Measured results of ε_r and $\tan\delta$ for $(Z_rS_n)T_iO_4$ ceramic rods at 10.4 GHz.

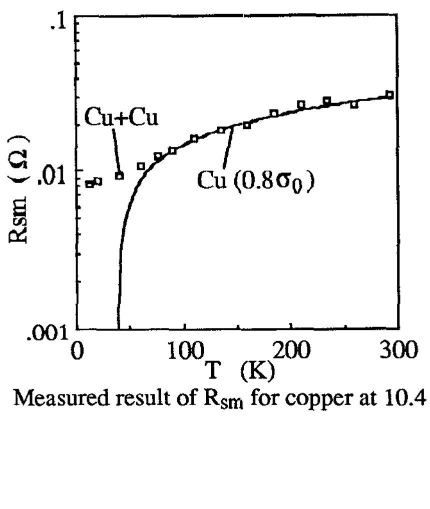


Fig. 5. Measured result of R_{sm} for copper at 10.4 GHz.

of Δf is given by the difference between the curve A' and the closed squares.

The values of R_s and X_s can be obtained from (12) using the Q_c and Δf values described above. These results are shown in Fig. 9. In the range of T>92 K, X_s agrees with R_s, while X_s is greater than R_s for T<92 K. In addition δ and λ are obtained from these R_s and X_s values using (10), as shown in Fig. 10. Similarly, using another dielectric rod of D=8.8 mm, we measured Z_s at 8.7 GHz [7].

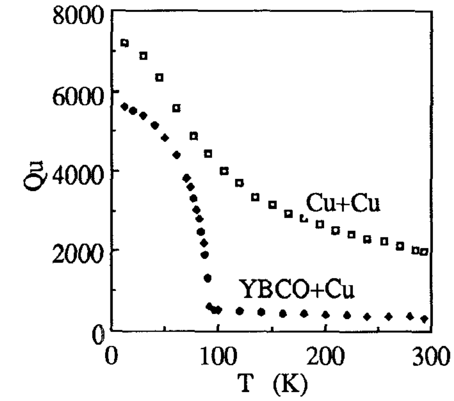


Fig. 6. Measured Q_u results for TE_{011} dielectric rod resonators in two cases of copper-copper plates (Cu+Cu) and YBCO-copper plates (YBCO+Cu).

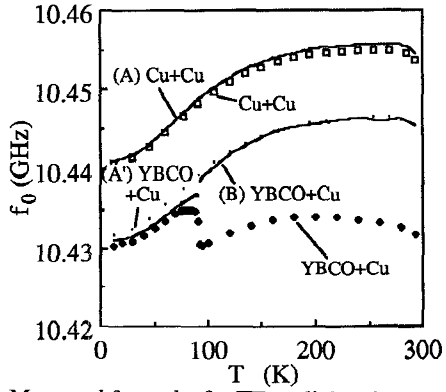


Fig. 7. Measured f_0 results for TE_{011} dielectric rod resonators in the copper-copper and YBCO-copper cases.

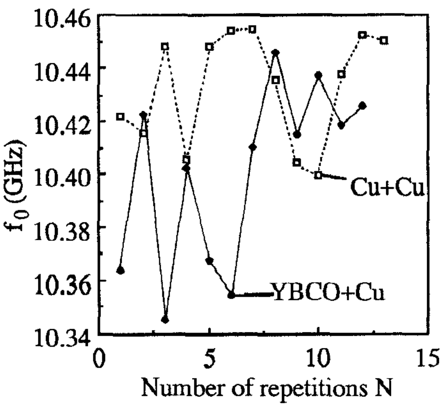


Fig. 8. The f_0 measurement repeated for TE_{011} dielectric rod resonators in the copper-copper and YBCO-copper cases at room temperature.

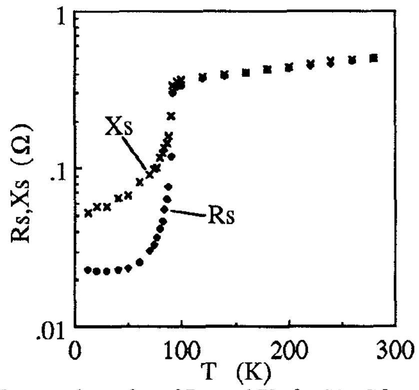


Fig. 9. Measured results of R_s and X_s for YBCO at 10.4 GHz. (Dielectric resonator method)

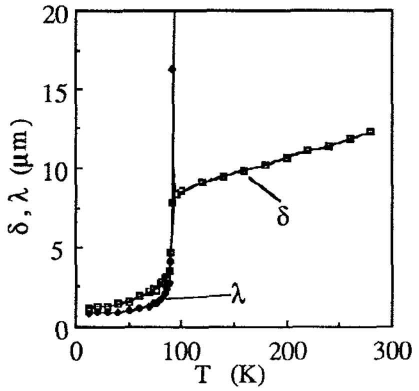


Fig. 10. Measured results of δ and λ for YBCO at 10.4 GHz. (Dielectric resonator method)

B. Cavity resonator method

Using a TE_{011} mode cavity resonator of L=8.11 mm, D=24.63 mm, and f_0=23.7 GHz, we measured the temperature dependence of f_0 and Q_u. In a similar way to the case of the dielectric resonator method, we obtained Z_s using (14). The measured results are shown in Fig. 11. These results of X_s have a much larger scatter than that for the dielectric resonator case.

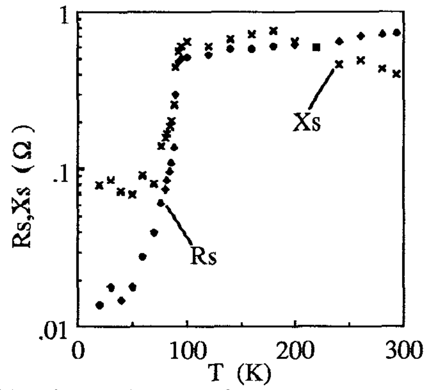


Fig. 11. Measured results of R_s and X_s for YBCO at 23.7 GHz.(Cavity method)

V. FREQUENCY DEPENDENCE OF Z_s

Using the measured results described above, we obtained the frequency dependences of R_s and X_s of the YBCO plate. These results are shown in Fig. 12, in which the broken lines indicate those for copper at 77 K. These measured results show that both R_s and X_s are proportional to $f^{0.5}$ even at T=77 K in the superconducting state as well as at T=200 K in the normal state, and do not agree with analysis of two-fluid model. To clarify this reason, we have a plan to perform similar measurements at some other frequencies.

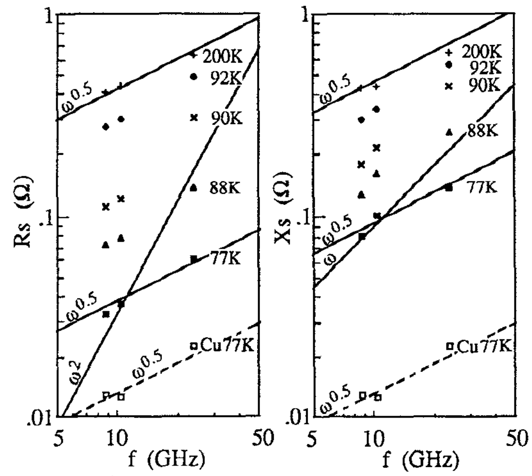


Fig. 12. Frequency dependences of R_s and X_s.

VI. CONCLUSION

Perturbation formulas for a TE_{011} mode dielectric resonator and for a TE_{011} mode cavity are derived to measure surface impedance Z_s of superconductor. Using these formulas, we measured Z_s of YBCO from 8.7 GHz to 23.7 GHz, as a function of temperature.

ACKNOWLEDGEMENTS

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